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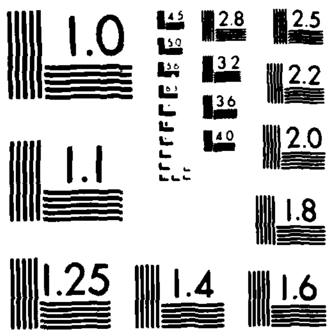

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THE K-PULSE AND RESPONSE WAVEFORMS FOR NON-UNIFORM TRANSMISSION LINES

by

E. M. Kennaugh, D. L. Moffatt, and N. Wang

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16. Abstract (Limit: 200 words)

Application of the K-pulse concept to a class of distributed-parameter systems which can be modelled by finite lengths of non-uniform transmission lines is demonstrated in this paper. The K-pulse of such a system is the excitation (input) waveform of finite duration which yields response waveforms of finite duration at all points of the system. Numerical techniques using a finite element method are developed to derive accurate approximation of the K-pulse and response waveforms for uniform and non-uniform transmission lines. Comparison is made with exact results to illustrate the accuracy and utility of the method.

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The research results given in this report represent, in large part, the last major work of Professor Emeritus Edward M. Kennaugh before his death. The report was completed by the coauthors who accept full responsibility for any misinterpretations or errors.

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I. INTRODUCTION

The purpose of this report is to illustrate the application of the K-pulse concept to a class of distributed-parameter systems which can be modelled by finite lengths of non-uniform transmission lines. The K-pulse of such a system is the excitation (input) waveform of finite duration which yields response waveforms of finite duration at all points of the system.

Numerical techniques using finite element methods are developed to derive accurate approximations of the K-pulse and response waveforms for uniform and non-uniform transmission lines. Comparison is made with exact results, where these can be obtained using other methods, to illustrate the accuracy and utility of the method.

II. THE R-MATRIX OF A TWO-PORT LINEAR SYSTEM

Let a two-port be represented symbolically as in Fig. 1, denoting traveling wave amplitudes (voltages) at ports 1 and 2 by a_i (inward traveling wave) and b_i (outward traveling wave). Then the scattering matrix (S) is defined:

$$\underline{b} = (S)\,\underline{a}\,, \qquad (1(a))$$

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \quad . \qquad (1(b))$$

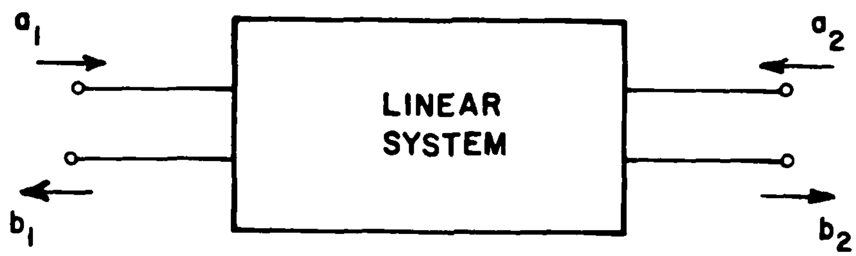

Figure 1. A linear two-port system.

When cascading two-ports, a more useful relation is given by

$$\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \begin{pmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{pmatrix} \begin{pmatrix} b_2 \\ a_2 \end{pmatrix} , \qquad (2)$$

a matrix which relates the right- and left-traveling wave amplitudes at port 1 to the corresponding right- and left-traveling amplitudes at port 2. If we denote this matrix by (R_{12}), we see that the matrix (R_{1n}) relating the wave amplitudes at the first port to those at the n^{th} port is just the product of n-1 matrices:

$$(R_{1n}) = (R_{12})\ (R_{23}) \ . \ . \ . \ (R_{n-1,n}) \ . \qquad (3)$$

To use the finite element method to analyze a non-uniform line, we first find the (R) matrix for a representative element, and then use matrix multiplication to find the resultant n-element approximation for the (R) matrix of the continuous line.

We note in passing that Dicke [1] defined the matrix in Eq. 2 as the T-matrix, but in order to avoid confusion with later usage of the term "T-matrix" to represent the perturbation matrix (S-I) [2], we shall adopt the notation of Kearns and Beatty [3], calling it the R-matrix.

The relation between (S) and (R) is

$$(R) = \frac{1}{s_{21}} \begin{pmatrix} 1 & -s_{22} \\ s_{11} & (s_{12}s_{21} - s_{11}s_{22}) \end{pmatrix} . \qquad (4)$$

and

$$(S) = \frac{1}{r_{11}} \begin{pmatrix} r_{21} & (r_{11}r_{22} - r_{12}r_{21}) \\ 1 & -r_{12} \end{pmatrix} . \qquad (5)$$

When entering and exiting lines in Fig. 1 (at ports 1 and 2) are assigned the same characteristic impedance, the scattering matrix (S) is symmetrical ($s_{12} = s_{21}$) and it follows that det (R) = 1, $r_{11}s_{12} = 1$. In such a case, if one "flips" ends of the two-port, exchanging right and left ends, the new (R) is given by

$$\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \begin{pmatrix} r_{11} & -r_{21} \\ -r_{12} & r_{22} \end{pmatrix} \begin{pmatrix} b_2 \\ a_2 \end{pmatrix} . \tag{6}$$

III. FINITE ELEMENT ANALYSIS OF LINE WITH DISTRIBUTED SHUNT CONDUCTANCE

It is customary to analyze distributed parameter networks by utilizing lumped constant elements, such as breaking a transmission line into lumped element T- or Pi-sections. However, we shall use delay elements corresponding to infinitesimal lengths of transmission line with fixed delay, which permits the final R-matrix of the system to be expressed in terms of polynomials in the variable $z = \exp(-2s\tau)$, where τ is the element delay.

Referring to Fig. 2, let the continuous shunt loading be modeled by N sections of loss-less line, each of length $\Delta L = L/N$, with a lumped shunt conductance G_n for the n^{th} section. We shall adopt the symbol d_n to indicate the amplitude (voltage) of the wave traveling to the right (dextra) at the left end of the n^{th} section, and the symbol s_n to denote the amplitude of the wave traveling to the left (sinistra) at the same ref ence plane. For a typical section:

$$\begin{pmatrix} d_n \\ s_n \end{pmatrix} = (R_n) \begin{pmatrix} d_{n+1} \\ s_{n+1} \end{pmatrix} , \qquad (7)$$

$$(R_n) = \begin{pmatrix} e^{s\tau} & 0 \\ 0 & e^{-s\tau} \end{pmatrix} \begin{pmatrix} 1+w_n & w_n \\ -w_n & 1-w_n \end{pmatrix} , \qquad (8)$$

where $w_n = G_n/2Y_o$, one-half the normalized conductance of the n^{th} shunt load. Using the z variable, we can define the matrix $(\bar{R}_n)$:

$$(R_n) = z^{-1/2}\,(1+w_n)\,(\bar{R}_n) \quad , \qquad (9)$$

where

$$(R_n) = \begin{pmatrix} 1 & P_n \\ -P_n z & (1-2p_n)z \end{pmatrix} \quad , \qquad (10)$$

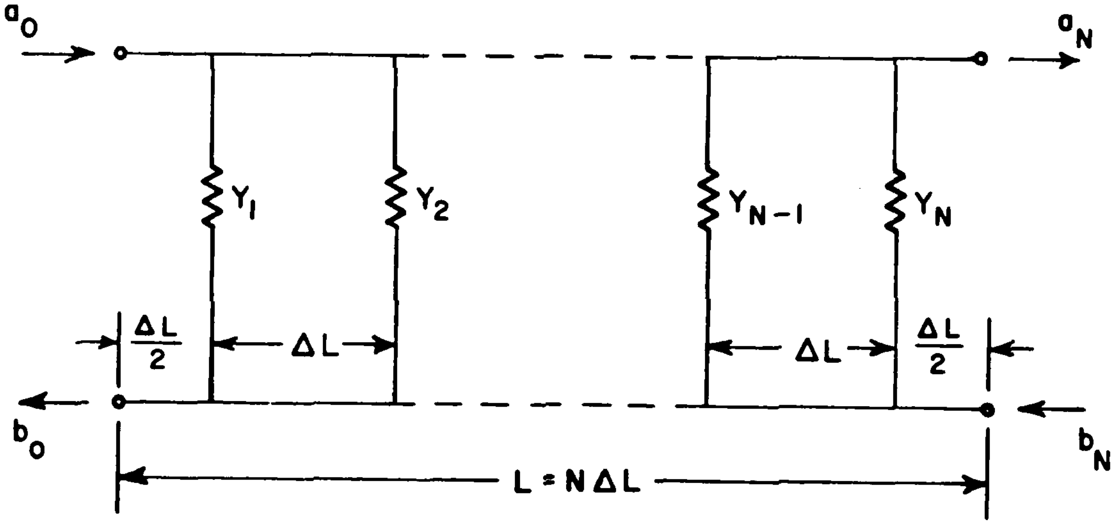

Figure 2. N-section model of finite transmission line.

and $P_n = G_n(G_n + 2Y_0)^{-1}$. The relation between input and output planes of Fig. 2 now becomes

$$\begin{pmatrix} d_0 \\ s_0 \end{pmatrix} = (R) \begin{pmatrix} d_{N+1} \\ s_{N+1} \end{pmatrix} \quad . \tag{11}$$

$$(R) = \alpha\, z^{-N/2} \begin{pmatrix} 1 & p_0 \\ -p_0 & (1-2p_0) \end{pmatrix} (\bar{R}) \quad , \tag{12}$$

where $\alpha = \prod_{n=0}^{N} (1+w_n)$, and

$$(\bar{R}) = (\bar{R}_1)\ (\bar{R}_2)\ .\ .\ .\ (\bar{R}_N) = \begin{pmatrix} R_{11}(z) & R_{12}(z) \\ zR_{21}(z) & zR_{22}(z) \end{pmatrix} , \tag{13}$$

and the $R_{ij}(z)$ are polynomials in z of order N-1.

Now

$$(R) = \alpha z^{-N/2} \begin{pmatrix} P_{11}(z) & P_{12}(z) \\ P_{21}(z) & P_{22}(z) \end{pmatrix} , \tag{14}$$

where

$$\begin{aligned} P_{11}(z) &= R_{11}(z) + p_o z R_{21}(z) \\ P_{12}(z) &= R_{12}(z) + p_o z R_{22}(z) \\ P_{21}(z) &= -p_o R_{11}(z) + (1-2p_o) R_{21}(z) \\ P_{22}(z) &= -p_o R_{12}(z) + (1-2p_o) z R_{22}(z) \end{aligned} \tag{15}$$

Hence the $P_{ij}(z)$ are all polynomials in z of order N. Each represents the Laplace transform of a train of N + 1 equally-spaced pulses with a fixed net duration $T = 2N\,\tau = 2L/C$, <u>independent</u> of N; where C is the wave velocity on the unloaded line.

Let us interpret these 4 finite duration waveforms for the distributed parameter system, in the limit as $N \to \infty$.

From Eqs. 10 and 13, it is easily shown that if $s_{N+1} = 0$, or the exiting line in Fig. 2 is terminated in the characteristic impedance Z_o of the unloaded line, $P_{11}(z)$ is the transform of that special input waveform $P_{11}(t)$ of finite duration applied at the left end of the loaded line which produces (a) a single attenuated (by a factor $1/\alpha$) and delayed (by $T/2 = L/c$) impulse at the exiting terminal, and (b) a left reflected wave $P_{21}(t)$ with finite duration. If $P_{11}(t)$ is applied at the right end of the line of Fig. 2 with the left end terminated in Z_0, then (a) the exiting wave at the left is the same attenuated and delayed impulse as above, while (b) the right reflected wave produced is $-P_{12}(t)$, also of finite duration. We thus define:

$$
\begin{aligned}
P_{11}(t) &= \mathcal{L}^{-1}\{P_{11}(z)\} = \text{K-pulse of network} \\
P_{21}(t) &= \mathcal{L}^{-1}\{P_{21}(z)\} = \Gamma_L \text{ pulse of network} \\
-P_{12}(t) &= \mathcal{L}^{-1}\{P_{12}(z)\} = \Gamma_R \text{ pulse of network} \\
P_{22}(t) &= \mathcal{L}^{-1}\{P_{22}(z)\} = \lambda\text{-pulse (K-pulse under time reversal)} .
\end{aligned}
\tag{16}
$$

The property suggested for $P_{22}(t)$ means that the time-reversed from or $P_{22}(T-t)$ is the K-pulse for the same line with all dissipative elements replaced by negative equivalents, i.e., exchanging $-G_n$ for G_n. In a lossless system, we shall find that $P_{11}(t) = P_{22}(T-t)$.

While the properties above hold for the $P_{ij}(t)$ of the line-lumped approximate of any order N, we are interested in the limiting form of these as $N \to \infty$, and how accurately this limit may be extrapolated from a finite element model with N less than 50. These questions will be addressed in Section 6, where computational results for various N are compared with exact waveforms, derived for the uniformly loaded line in Appendix II.

Values of the shunt loads $G_n(N)$ must be determined before calculations for the finite element model are initiated. These values are derived in Appendix I for the line with uniform shunt conductance as well as the line with a linear taper of shunt conductance. For simplicity, we have to this point assumed entering and exiting lines in Fig. 2 have the same characteristic impedance as the unloaded line; the case where these differ can easily be accommodated by adding at most two simple R-matrices for junctions between dissimilar lines to the chain product in Eq. 12. This case will be discussed fully in Section 5.

IV. FINITE ELEMENT ANALYSIS OF LINE WITH NON-UNIFORM CHARACTERISTIC IMPEDANCE

A line with continuously varying characteristic impedance is modeled by N uniform line segments in cascade, the characteristic impedances of the discrete elements matching that of the line as a staircase function. If the phase velocity is non-uniform as well, the N sections will be of dissimilar lengths, but constant phase delay. We shall consider here the special case arising when a non-uniform line is used to model transmission and reflection by a dielectric slab with a varying dielectric constant. Referring to Fig. 3, the non-uniform line is represented by N uniform sections with Z_n, β_n the characteritic impedance and phase constant of the n^{th} section. The lengths $(\Delta L)_n$ of the sections are chosen such that $\beta_n (\Delta L)_n = \beta\Delta L =$ constant. Thus, the propagation time T/2 through the non-uniform slab is composed of N equal delays τ for each section where $T = 2N\tau$.

The R matrix of the n^{th} element of Fig. 3 is given by

$$\begin{pmatrix} d_n \\ s_n \end{pmatrix} = (R_n) \begin{pmatrix} d_{n+1} \\ s_{n+1} \end{pmatrix} , \tag{17}$$

where

$$(R_n) = \begin{pmatrix} e^{s\tau} & 0 \\ 0 & e^{-s\tau} \end{pmatrix} \begin{pmatrix} k_n & 1-k_n \\ 1-k_n & k_n \end{pmatrix} . \tag{18}$$

Using $z = \exp(-2s\tau)$, and

$$(R_n) = k_n \, z^{-1/2} \, (\bar{R}_n) \quad ;$$

$$(\bar{R}_n) = \begin{pmatrix} 1 & \Gamma_n \\ z\Gamma_n & z \end{pmatrix} \quad , \tag{19}$$

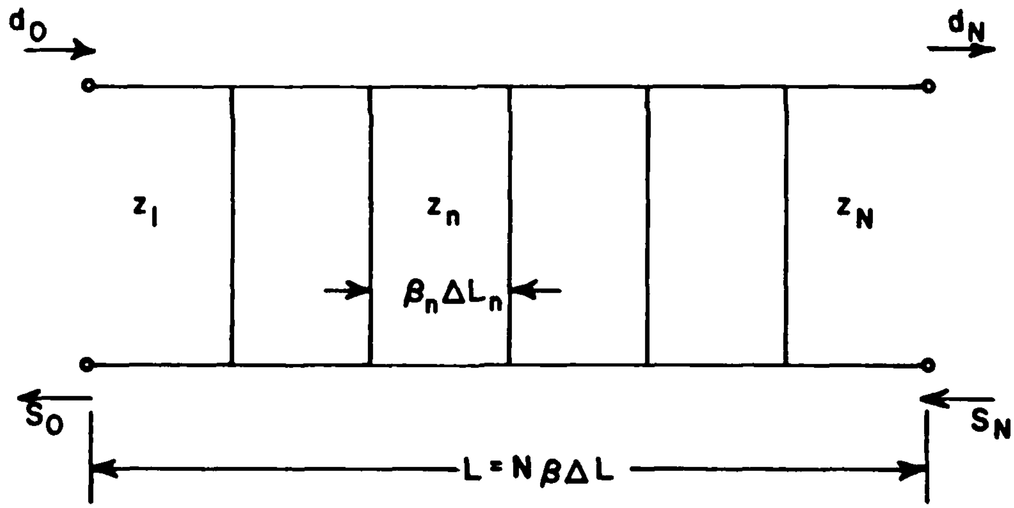

Figure 3. Lines with non-uniform characteristic impedance.

where $\Gamma_n = (Z_{n+1} - Z_n)/(Z_{n+1} + Z_n)$, $k_n = (1 + Z_n/Z_{n+1})/2$. As before (see Eqs. 10-14),

$$\begin{pmatrix} d_0 \\ s_0 \end{pmatrix} = (R) \begin{pmatrix} d_{N+1} \\ s_{N+1} \end{pmatrix} \quad , \qquad (20)$$

where

$$(R) \quad = \quad \alpha\, z^{-N/2} \begin{pmatrix} 1 & \Gamma_0 \\ \Gamma_0 & 1 \end{pmatrix} (\bar{R}) \quad , \qquad (21)$$

and $\alpha = \prod_{n=0}^{N} k_n$.

$$(\bar{R}) \quad = \quad (\bar{R}_1)\ (\bar{R}_2)\ .\ .\ .\ (\bar{R}_N) \quad = \begin{pmatrix} R_{11}(z) & R_{12}(z) \\ zR_{21}(z) & zR_{22}(z) \end{pmatrix} . \qquad (22)$$

Since the $R_{ij}(z)$ are polynomials in z of order N-1,

$$(R) = \alpha z^{-N/2} \begin{pmatrix} P_{11}(z) & P_{12}(z) \\ P_{21}(z) & P_{22}(z) \end{pmatrix} , \tag{23}$$

where

$$\begin{aligned} P_{11}(z) &= R_{11}(z) + \Gamma_0 z R_{21}(z) \\ P_{12}(z) &= R_{12}(z) + \Gamma_0 z R_{22}(z) \\ P_{21}(z) &= \Gamma_0 R_{11}(z) + z R_{21}(z) \\ P_{22}(z) &= \Gamma_0 R_{12}(z) + z R_{22}(z) \end{aligned} \tag{24}$$

As in the previous section, the $P_{ij}(z)$ are all polynomials in z of order N; the interpretation of the 4 finite duration waveforms, each of fixed net duration $2N\tau = T$, which are the inverse Laplace transforms of the $P_{ij}(z)$ follows as in Section 3, preceding.

The values of $\Gamma_n(N)$ and $k_n(N)$ must be determined from the particular dielectric constant variation of interest before calculations of the N^{th} order approximants are initiated. These values are derived in Appendix II for the line representing linear variation of ε_n in a dielectric slab. In Section 6, the convergence of the waveforms as N increases will be examined.

V. TERMINATION EFFECTS

In Section 3 and 4, the discrete element model was employed to approximate continuous variation of conductivity or impedance of a non-uniform line. Discontinuous changes in line parameters such may arise at exiting and entering ports were not considered. In Section 3, it was assumed that the line with distributed conductance was inserted between lines of the same characteristic impedance with unloaded line. The resulting R matrix of the system has unit determinant. If entering and exiting lines have characteristic impedance Z_s and Z_d respectively, the R matrix for the system is given by (R_T).

$$(R_T) = k_d \;\; k_d \begin{pmatrix} 1 & \Gamma_s \\ \Gamma_s & 1 \end{pmatrix} (R) \begin{pmatrix} 1 & \Gamma_d \\ \Gamma_d & 1 \end{pmatrix} , \qquad (25)$$

where

$$k_s = (1 + Z_o/Z_s)/2,\ \Gamma_s = (Z_o - Z_s)/(Z_o + Z_s)\ ,$$

$$k_d = (1 + Z_d/Z_o)/2 \text{ and}$$

$$\Gamma_d = (Z_d - Z_o)/(Z_d + Z_o)\ .$$

When $Z_s = Z_d$, the determinant of the matrix (R_T) has unit determinant, a property which holds whenever entering and exiting lines have a common characteristic impedance. In Section 4, the characteristic impedance of entering and exiting lines were not generally the same, so that the resultant R matrix of Eq. 2 does not have unit determinant, but a value of $\frac{Z_{N+1}}{Z_0}$. If the limiting input and exiting impedances of the line of Fig. 3 are Zo and Zn+1, respectively, then we can obtain the R_T for new entering and exiting impedances Zs, Zd, respectively, by Eq. 24 where

$$k_s = (1 + Z_0/Z_s)/2, \ \Gamma_s = (Z_0 - Z_s)/(Z_0 + Z_s),$$

$$k_d = (1 + Z_d/Z_{N+1})/2, \text{ and } \Gamma_d = (Z_d - Z_{N+1})/(Z_d + Z_{N+1}).$$

The resultant (R_T) will have a determinant of Z_d/Z_s. If $Z_d = Z_s$, entering and exiting lines have common characteristic impedance, then the resulting (R_T) will have unit determinant.

With reference to Fig. 4, let the R-matrix of the distributed parameter network referred to entering and exiting line characteristic impedance Z_s and Z_d be derived as outlined in Eq. 24. Then the K-pulse of the system with load Z_d and generator impedance Z_s is given as

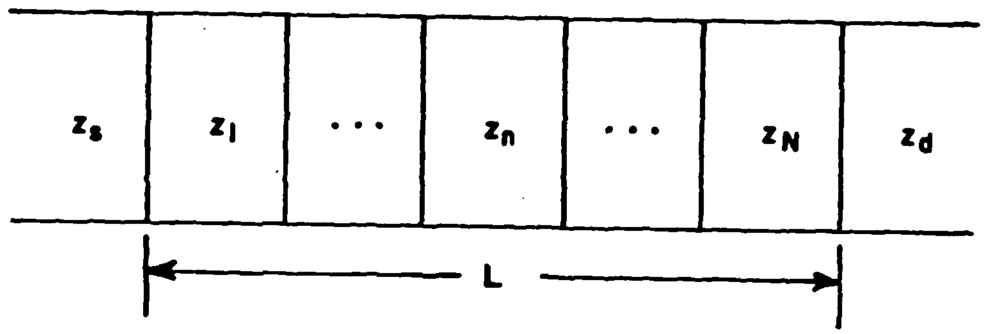

Figure 4. A finite line inserted between entering line with Z_s and exiting line with Z_d.

$K(t) = \mathcal{L}^{-1}\{P_{11}(z) + \Gamma_d P_{12}(z) + \Gamma_s P_{21}(z) + \Gamma_d \Gamma_s P_{22}(z)\}$ where $\Gamma_d = (Z_d - Z_L)/(Z_d + Z_L)$ and $\Gamma_s = (Z_L - Z_s)/(Z_L + Z_s)$. Similar expressions can be obtained for other response waveforms. Numerical results of K-pulse and reflected pulses for a finite transmission lines inserted between lines with various Z_s and Z_d will be illustrated in Sec. 6.

VI. EXAMPLES

To illustrate simple examples of the K-pulse in a distributed parameter system, we model the reflection of a plane wave at normal incidence to a planar dielectric slab by a network in which a length L of line with conductance G is inserted between two semi-infinite lines with characteristic impedance Z_s and Z_d. The K-pulse and reflected waveforms are then calculated using a N-section line-lumped approximation to the finite transmission line. In the following, various numerical results obtained for the approximations of the K-pulse and response waveforms for uniform and nonuniform lines are presented. Comparison is made with exact results, where these can be obtained using other methods, to illustrate the accuracy and utility of the methods.

A. UNIFORM CONDUCTANCE G

Consider first a lossless line with length L shorted at one end and appended to a semi-infinite line of twice the intrinsic impedance, corresponding to a slab with dielectric constant $\varepsilon_r = 4$. Thus, $G = 0$, $Z_s = 2Z_0$ and $Z_d = 0$, where Z_0 is the intrinsic impedance of the lossless slab.

In Fig. 5, the K-pulse consists of a unit impulse followed by a second impulse of 1/3 with a delay of 2L/C, corresponding to the transit time down and back the shorted line. The reflected waveform is the time reversed negative of the K-pulse. If uniform shunt conductance loading

G is now introduced along with shorted line, such that the total conductance in the finite line equals the surge admittance, the K-pulse and reflected pulse are shown in Fig. 6. As in the lossless case, a unit impulse begins the input. Thereafter, signals returning to the input junction and reflected down the line segment are cancelled or "killed" by subsequent input contributions until the final signal reflected from the shorted end is returned. Since no further signal travels down the line segment, the shorted section is at rest after $\Delta t = 2L/C$. The K-pulse or "kill-pulse," has a duration equal to the round trip time.

The waveforms of Fig. 6 are calculated using a N-section line-lumped approximation to the uniform line. For comparison, the exact results are also presented in Fig. 6. It is observed that the K-pulse and reflected waveform converge rapidly to the exact results as N increases from N = 10 to N = 40.

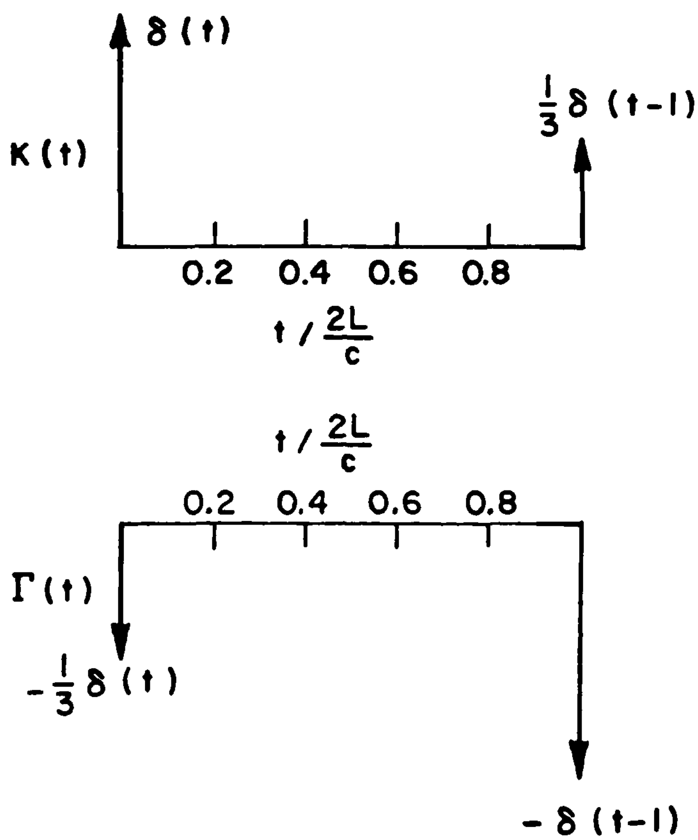

Figure 5. Lossless grounded slab.

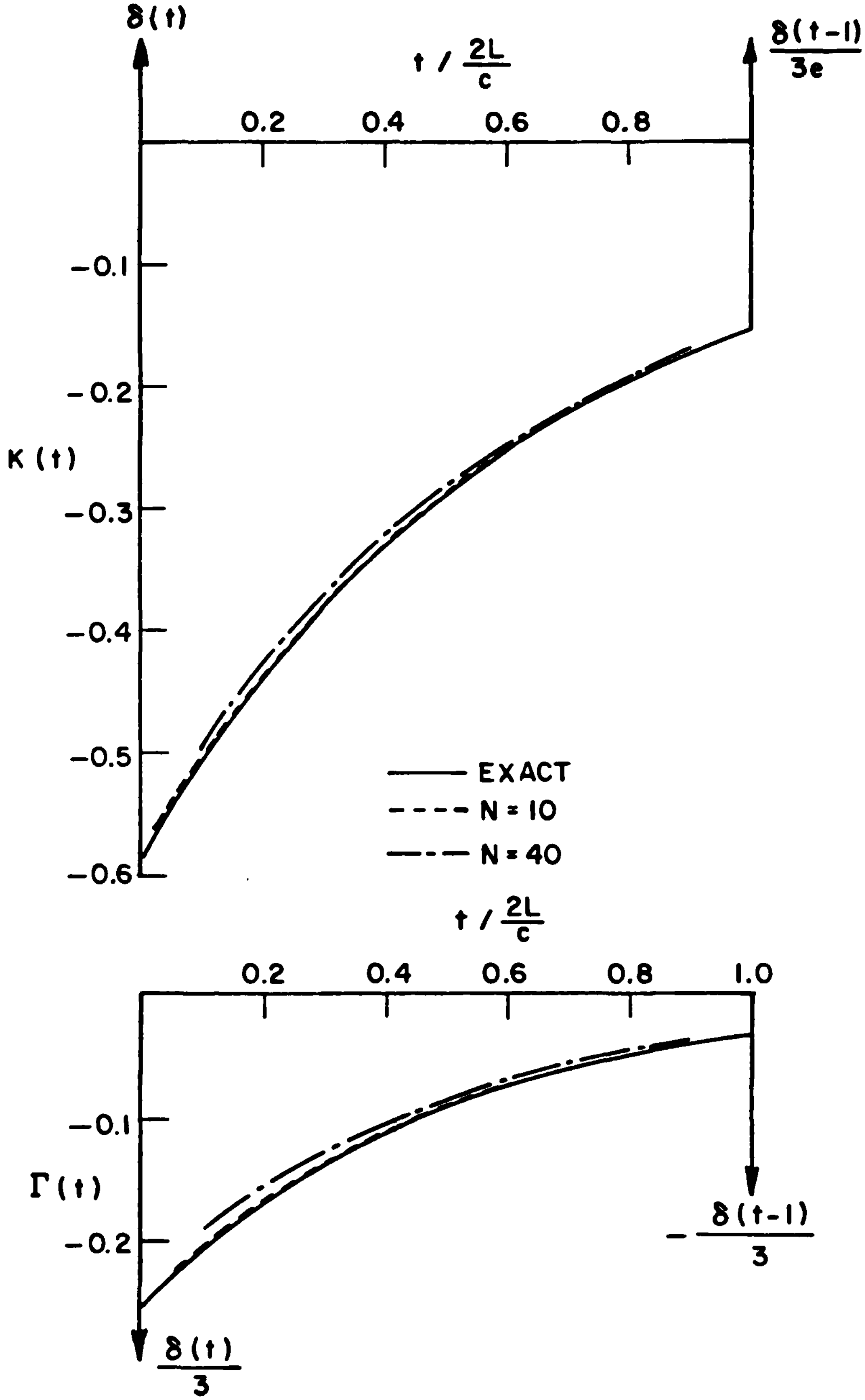

Figure 6. Shorted uniform lossy slab.

Fig. 7 and Fig. 8 present the K-pulse and reflected pulse for a uniform lossy line inserted between two semi-infinite lines of different characteristic impedance, corresponding to a planar slab separating two half spaces with characteristic impedances Z_s and Z_d respectively. Fig. 7 shows the results for the symmetrical case ($Z_s = Z_d = 2Z_o$), where Z_o is the intrinsic impedance of the unloaded line. It is again observed that as N increases from 10 to 40, the N-section line-lumped approximation rapidly converges to the exact results. Note that for the symmetrical case, the left-reflected and right-reflected pulse are identical. For the asymmetrical case shown in Fig. 8 ($Z_s = 2Z_o$, $Z_d = Z_o$), though, the left and right-reflected waveforms are not the same. In both cases, the K-pulse and response waveforms have a duration time equal to the round-trip transit time.

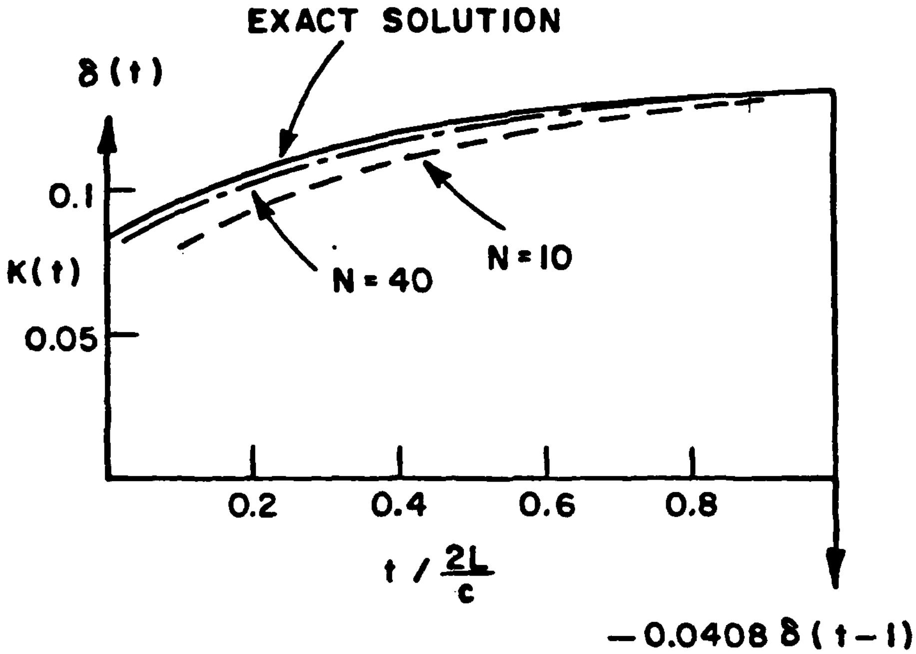

Figure 7a. K-pulse and reflected waveforms for uniform lossy slab ($Z_s = Z_d = 2.0$, $k = 1$)

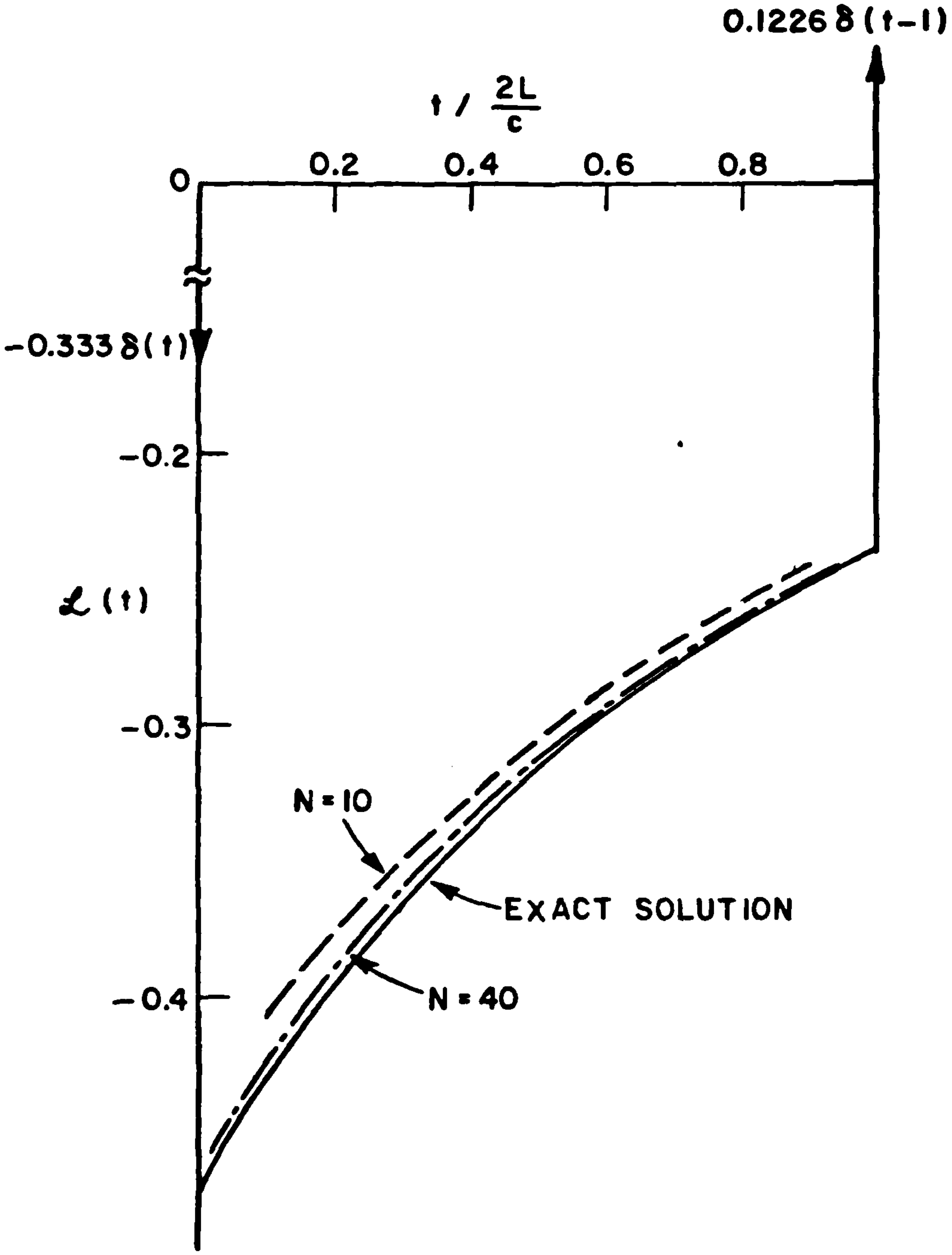

Figure 7b. K-pulse and reflected waveforms for uniform lossy slab ($Z_s = Z_d = 2.0$, $k = 1$)

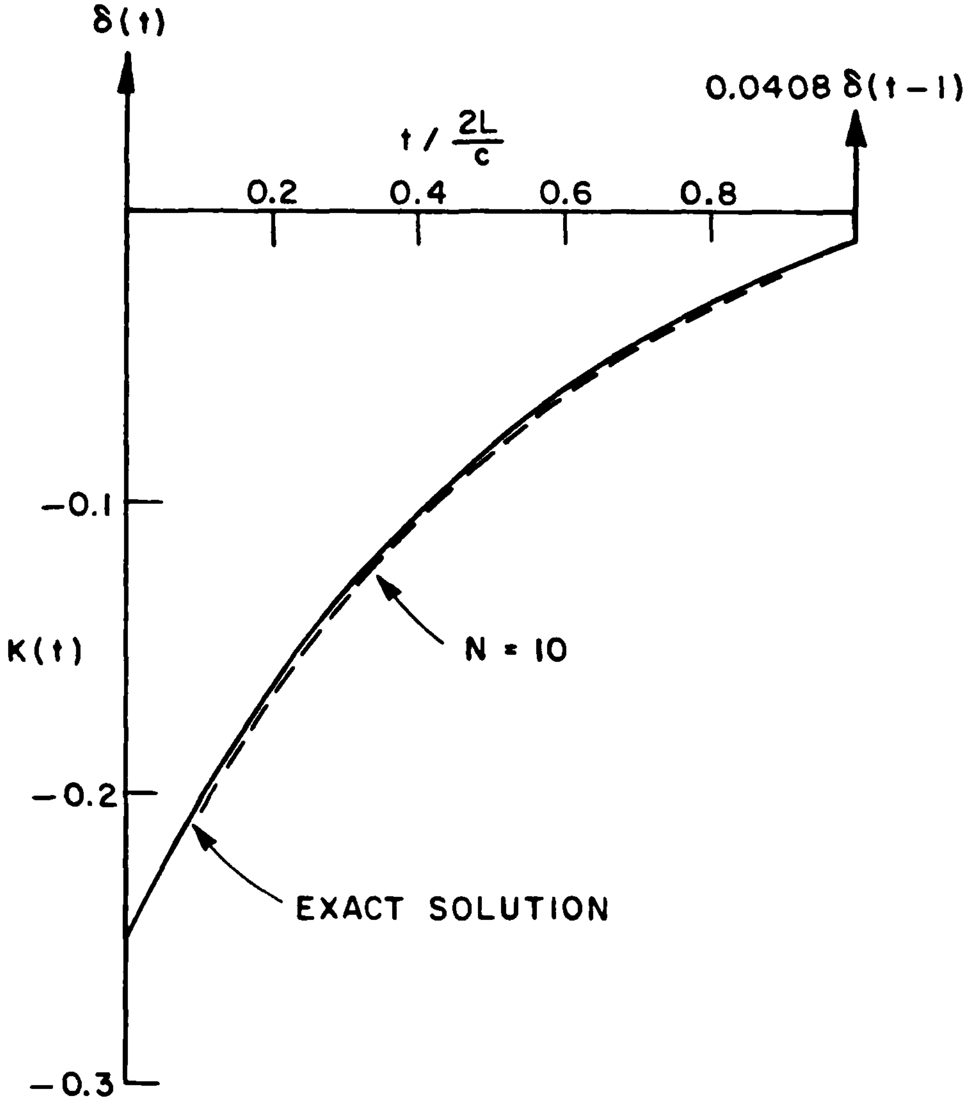

Figure 8a. K-pulse and reflected waveforms for uniform lossy slab (Z_s = 2.0, Z_d = 0.5, k = 1)

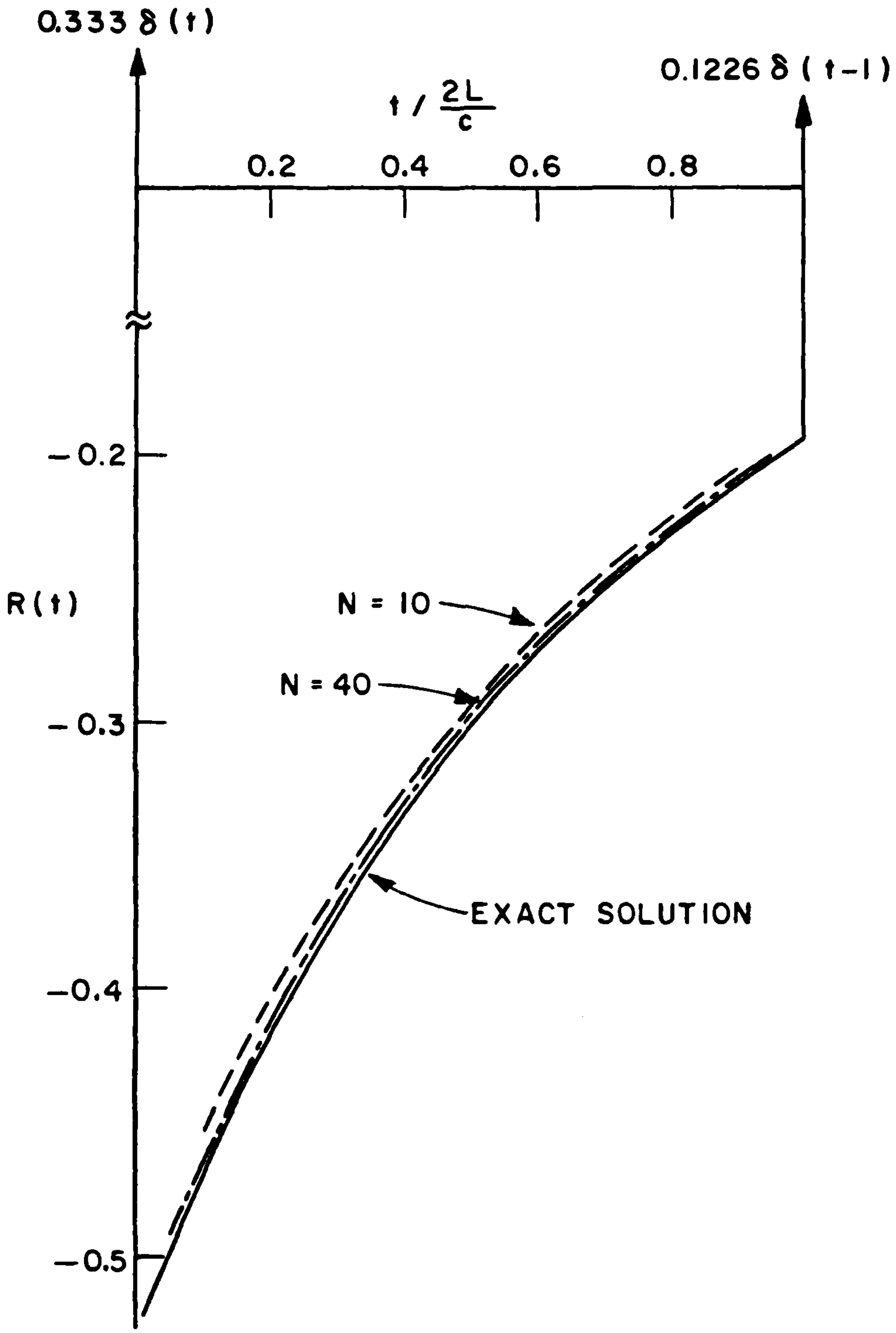

Figure 8b. K-pulse and reflected waveforms for uniform lossy slab (Z_s = 2.0, Z_d = 0.5, k = 1)

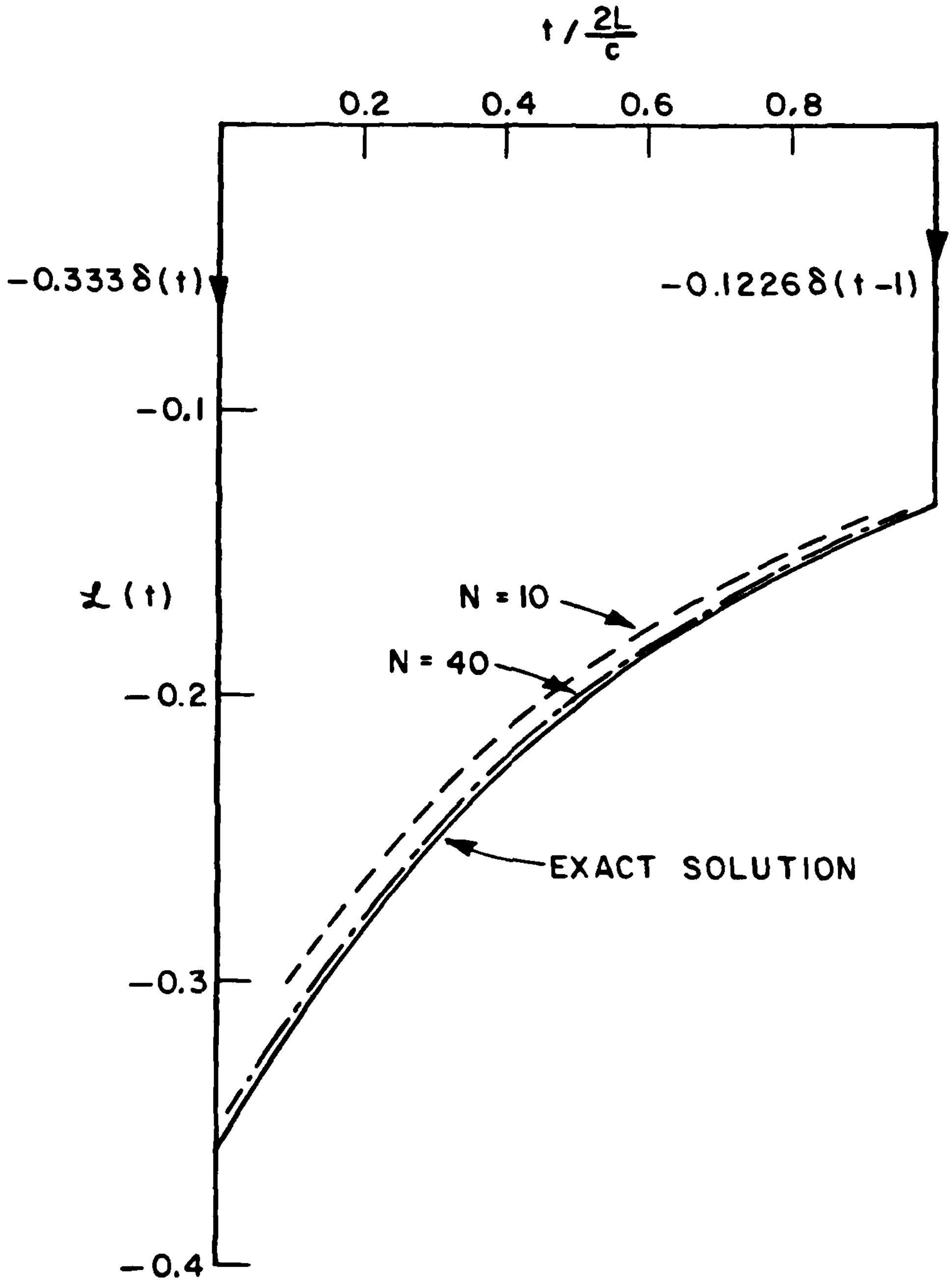

Figure 8c. K-pulse and reflected waveforms for uniform lossy slab (Z_s = 2.0, Z_d = 0.5, k = 1)

B. LINEARLY VARYING CONDUCTANCE G

Consider a finite line with linearly tapered shunt conductance, with the total shunt conductance equal to the intrinsic impedance of the unloaded line. Again, the continuous shunt loading is modeled by N sections of lossless line, each of length $\Delta L = L/N$, with a lumped shunt conductance G_n for the n^{th} section. Values of the shunt loads G_n are determined in Appendix II. Figs. 9-11 present the K-pulse and response waveforms for a linearly tapered line with various terminations. It is observed that convergence to the K-pulse and response waveforms are rapid and simple, even for lines with continuously varying parameters.

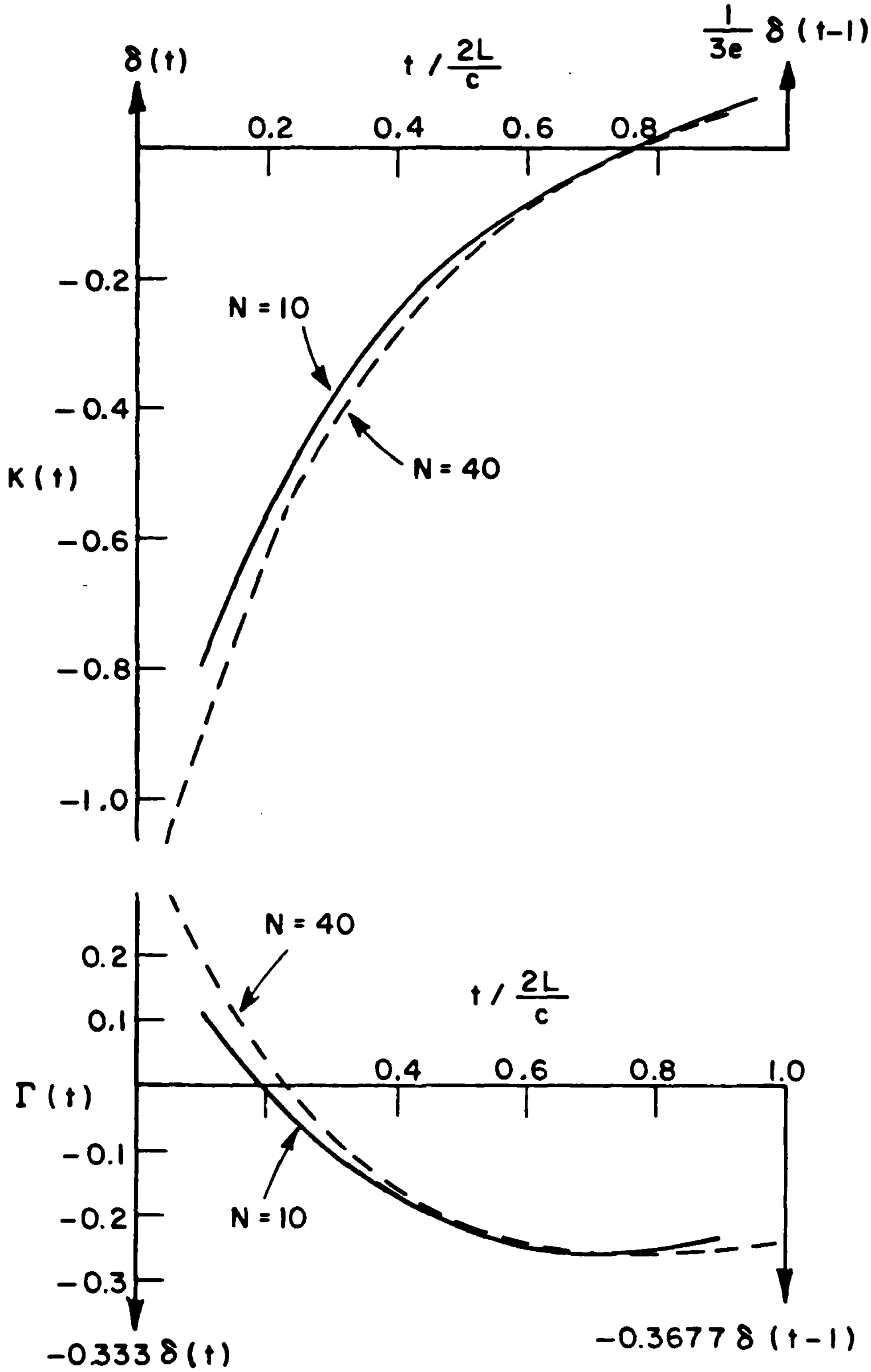

Figure 9. K-pulse and reflected waveforms for shorted tapered line (Z_s = 2.0, Z_d = 0.0, k = 1)

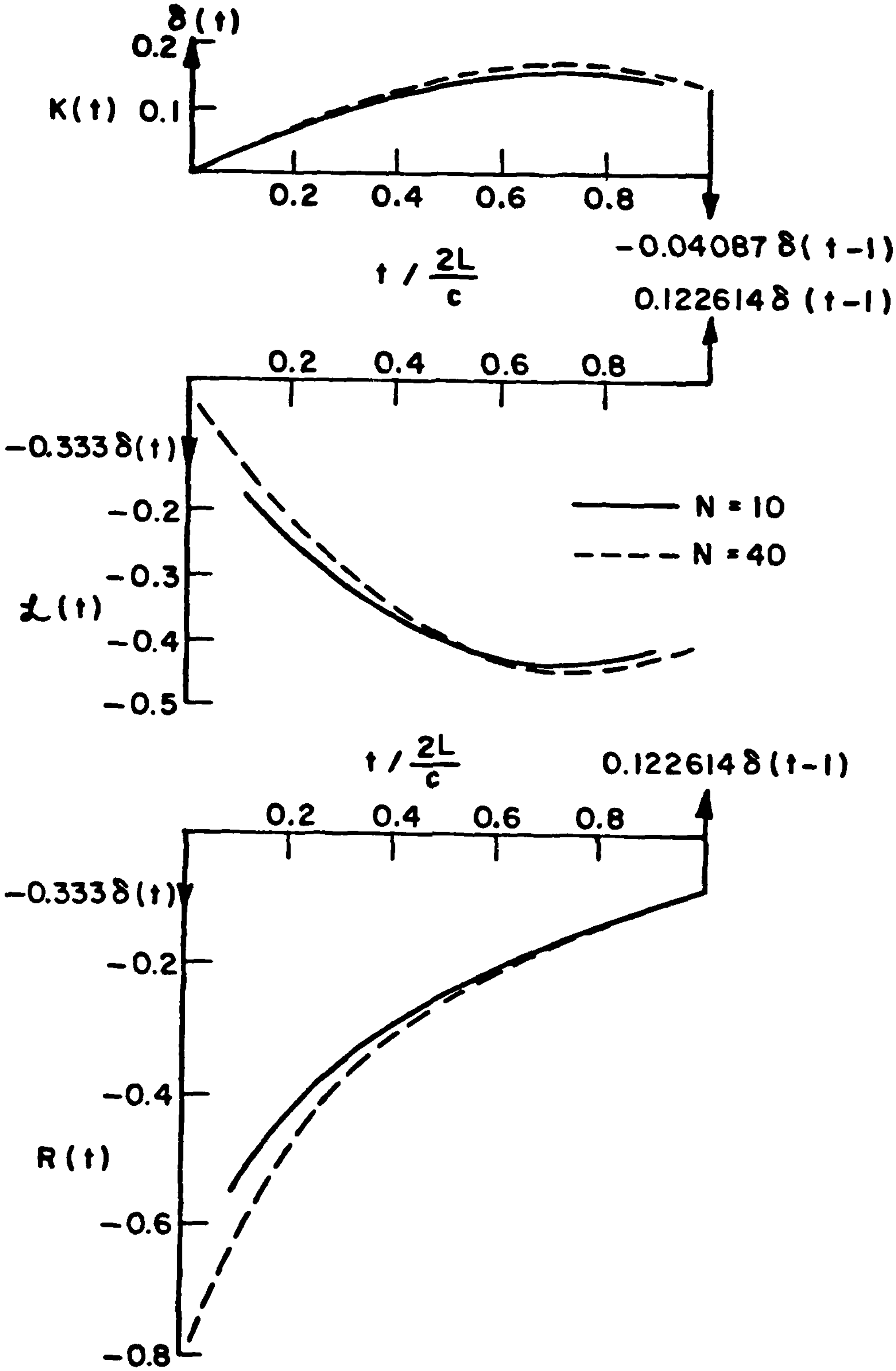

Figure 10. K-pulse and reflected waveforms for tapered lossy slab ($Z_s = Z_d = 2.0$, $k = 1$)

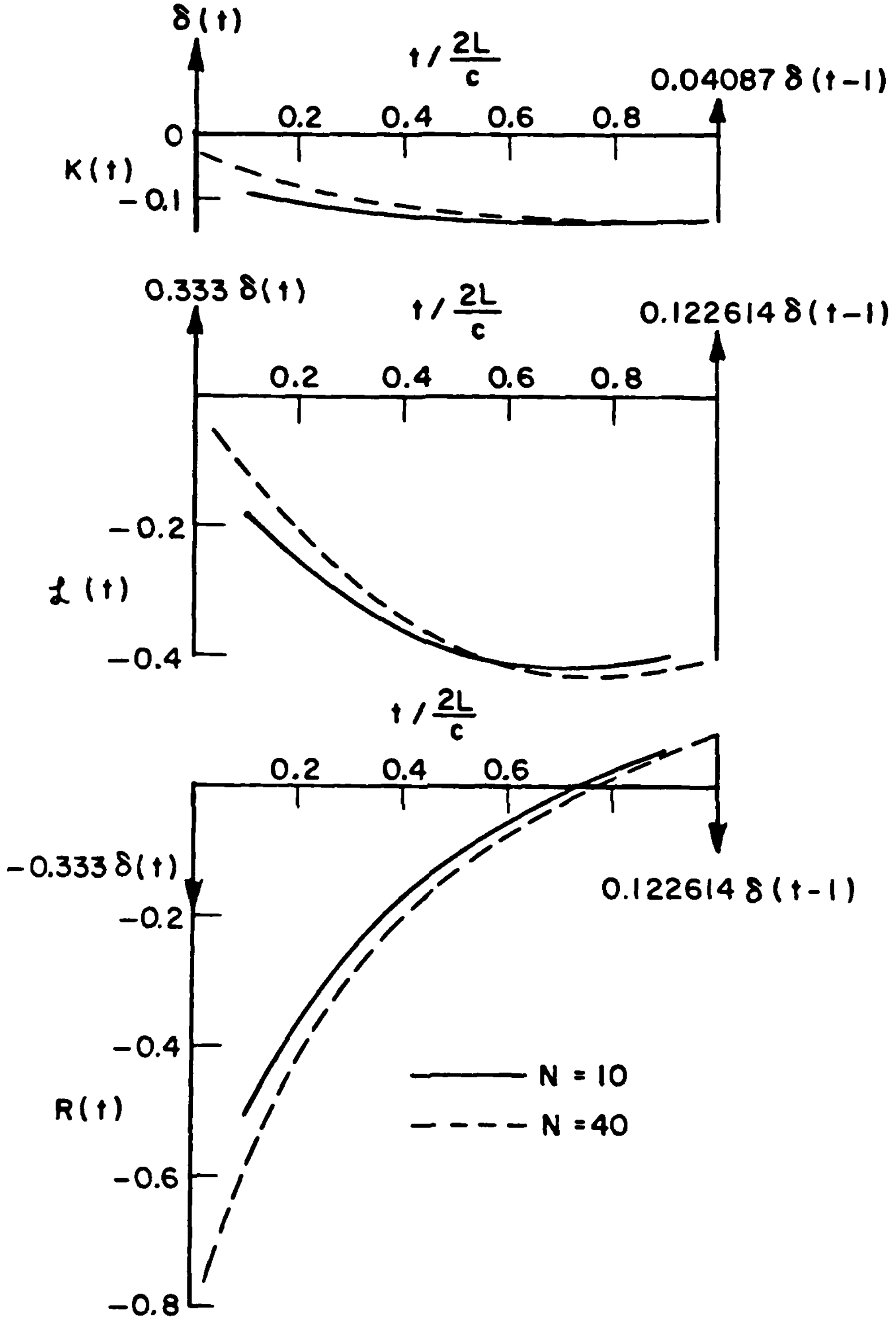

Figure 11. K-pulse and reflected waveforms for tapered lossy slab ($Z_s = 0.5$, $Z_d = 2.0$, $k = 1$)

C. LINEAR VARIATION OF CHARACTERISTIC IMPEDANCE

Numerical results of K-pulse and reflected waveforms for a line with continuously varying characteristic impedance are presented in this section. Waveforms are obtained using a model with N uniform line segments in cascade, the characteristic impedances of the discrete elements matching that of the line as a staircase function. If the phase velocity is non-uniform as well, the N sections will be of dissimilar lengths, but constant phase delay. We shall consider here the special case arising when a non-uniform line is used to model transmission and reflection by a planar dielectric slab with a linearly varying dielectric constant. The profile of the dielectric constant ε_r is shown in Fig. 12. The two half spaces separated by the slab have a dielectric constant ε_s and ε_d respectively.

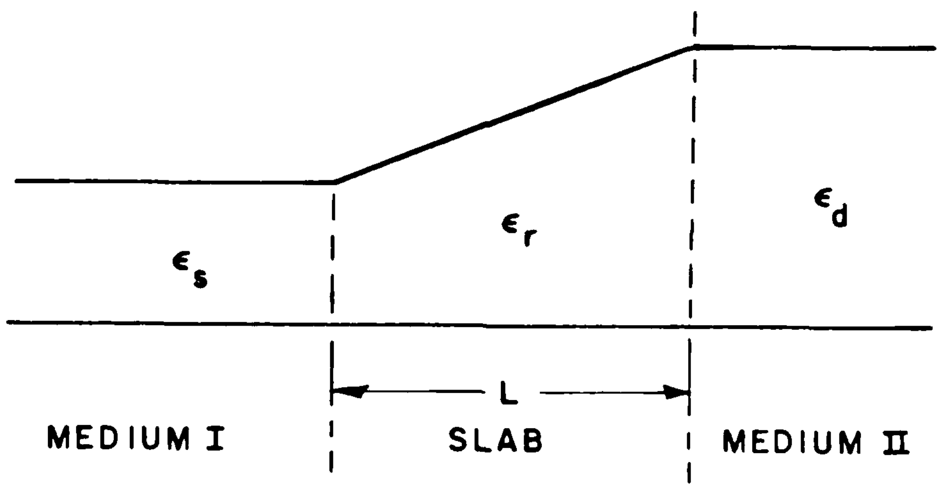

Figure 12. Profile of the dielectric constant for a planar dielectric slab

Fig. 13 shows the K-pulse and reflected pulse for a planar slab with a dielectric constant ε_r, linearly tapered from $\varepsilon_s = \varepsilon_0$ to $\varepsilon_d = 2\varepsilon_0$. The waveforms shown in Fig. 13 are calculated using a 20-section line-lumped approximation to the planar slab. Calculations using N = 40 shows that convergence to the K-pulse and reflected waveform is rapid and simple.

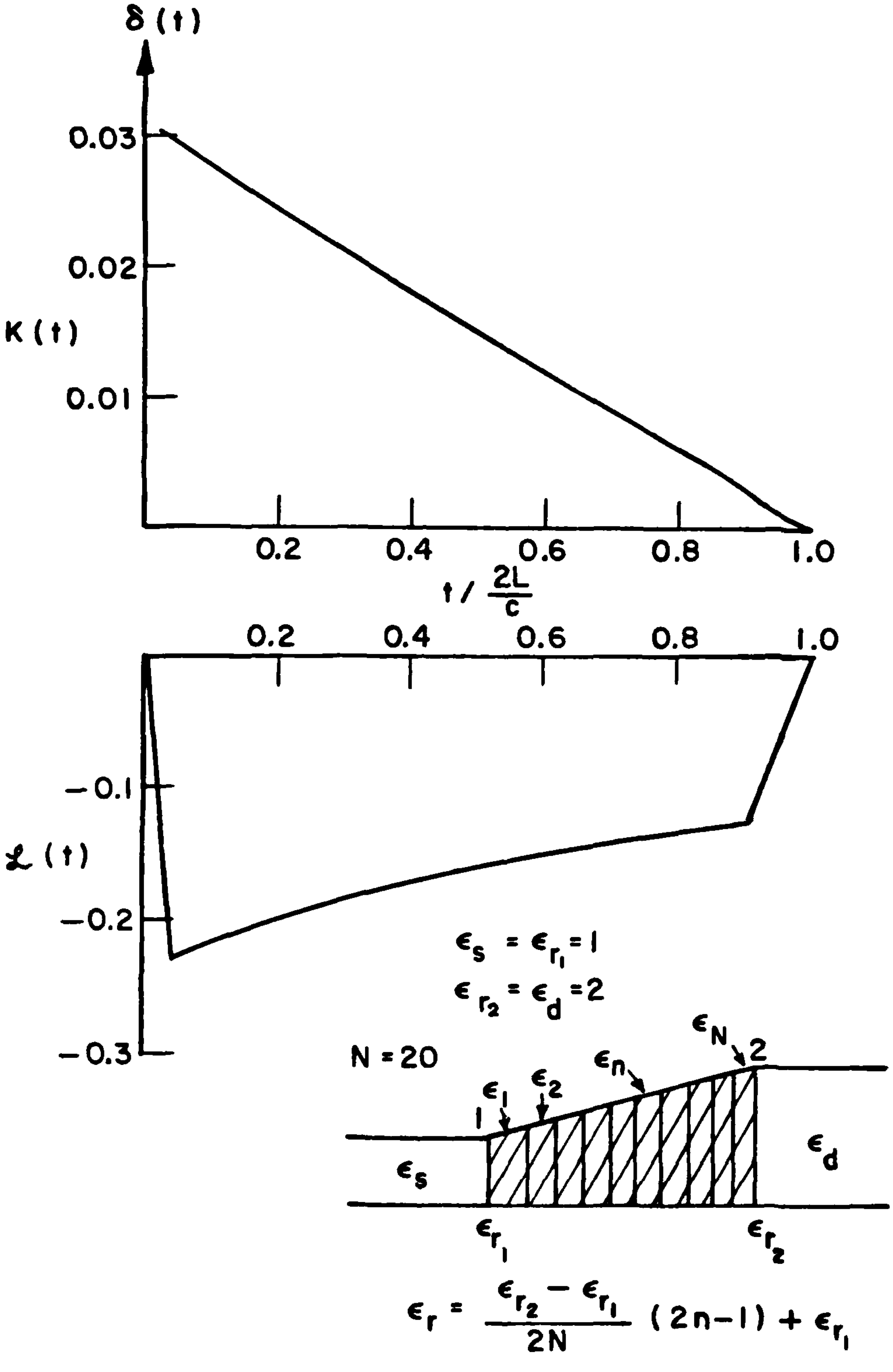

Figure 13. K-pulse for line with continuously varying ϵ_r

VII. CONCLUSIONS

In this report we illustrate the application of the K-pulse concept to a class of distributed-parameter systems which can be modelled by finite lengths of non-uniform transmission lines. The K-pulse of such a system is the excitation (input) waveform of finite duration which yields response waveforms of finite duration at all points of the system. Numerical techniques using finite element methods are developed to derive accurate approximations of the K-pulse and response waveforms for uniform and non-uniform transmission lines. Comparison is made with exact results derived for the uniformly loaded line, to illustrate the accuracy and utility of the method.

The next logical step in this analysis is to address the inverse problem, i.e., given the K-pulse and response waveforms, what are the electrical parameters of the line. Kennaugh had claimed in an earlier report [4] that synthesizing the parameters of the non-uniform line from measured K-pulse and response waveforms was as equally tractable as the direct problem. The key role of the K-pulse in factoring the system before attempting synthesis has been clearly established in this approach, which differs from the one-dimensional inversion techniques. It is intended to investigate this problem as time and funds permit.

APPENDIX I

K-pulse for Uniform Lossy Line with Short-Circuit Termination.

Reflection by a lossy distributed-parameter network furnishes a useful example for application of K-pulse concepts. As shown in Fig. A-1, a length l of lossy line with characteristic impedance Z_1 is shorted at the far end. The reflection coefficient (voltage) at the input terminal when connected to a uniform line of characteristic impedance Z_0 is of interest.

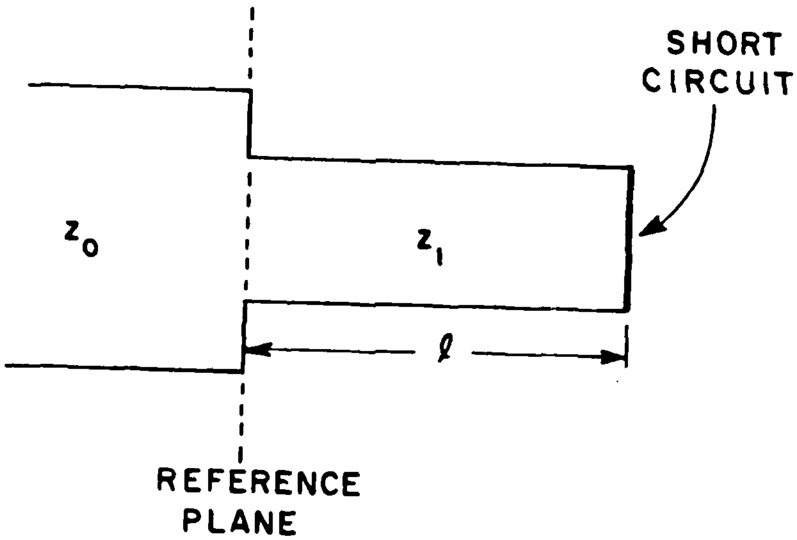

Figure A-1. Lossy line configuration

We assume that line loss is modeled by a uniformly distributed shunt conductance, such that the total shunt conductance over the length l is a specified fraction of the surge admittance of the line without loss, i.e.

$$\frac{\text{total } G_{shunt}}{Y_1} = K \quad .$$

The ratio of the surge admittance of the entire line (to the left of reference plane) to that of the line is also a specified parameter:

$$\mu = \frac{Y_0}{Y_1} \quad .$$

The expression for the voltage reflection coefficient can be derived as:

$$\Gamma_v = \frac{\mu \tanh \left(S\tau \sqrt{\frac{K + S\tau}{S}}\right) - \sqrt{\frac{K + S\tau}{S}}}{\mu \tanh \left(S\tau \sqrt{\frac{K + S\tau}{S\tau}}\right) + \sqrt{\frac{K + S\tau}{S\tau}}} \quad ,$$

where $S = j\omega\tau$, $\tau = \frac{\ell}{C_1}$ and C_1 is the velocity of a wave traveling the finite line section in the absence of loss.

Recognizing that

$$\tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}} ,$$

the voltage reflection coefficient Γ_v can be rewritten as

$$\Gamma_v = \frac{\left[\mu\sqrt{\frac{S\tau}{S\tau+K}} - 1\right] e^{S\tau\sqrt{\frac{S\tau+K}{S\tau}}} - \left[\mu\sqrt{\frac{S\tau}{S\tau+K}} + 1\right] e^{-S\tau\sqrt{\frac{S\tau+K}{S\tau}}}}{\left[\mu\sqrt{\frac{S\tau}{S\tau+K}} + 1\right] e^{S\tau\sqrt{\frac{S\tau+K}{S\tau}}} - \left[\mu\sqrt{\frac{S\tau}{S\tau+K}} - 1\right] e^{-S\tau\sqrt{\frac{S\tau+K}{S\tau}}}} .$$

Multiply numerator and denominator by

$\frac{e^{-S\tau}}{\mu}$, one obtains

$$\Gamma v = \frac{\left(\frac{S\tau}{\sqrt{S\tau(S\tau+K)}} - \frac{1}{\mu}\right) e^{\sqrt{S\tau(S\tau+K)} - S\tau} - \left(\frac{S\tau}{\sqrt{S\tau(S\tau+K)}} + \frac{1}{\mu}\right) e^{-\sqrt{S\tau(S\tau+K)} - S\tau}}{\left(\frac{S\tau}{\sqrt{S\tau(S\tau+K)}} + \frac{1}{\mu}\right) e^{\sqrt{S\tau(S\tau+K)} - S\tau} - \left(\frac{S\tau}{\sqrt{S\tau(S\tau+K)}} - \frac{1}{\mu}\right) e^{-\sqrt{S\tau(S\tau+K)} - S\tau}} ,$$

with a change of variable, $\overline{S} = S\tau + a$, where $a = K/2$, Γ_v can be written as:

$$\Gamma v = \frac{\left(\frac{\bar{s}-a}{\sqrt{\bar{s}^2-a^2}} - \frac{1}{\mu}\right) e^{\sqrt{\bar{s}^2-a^2}} - \left(\frac{\bar{s}-a}{\sqrt{\bar{s}^2-a^2}} + \frac{1}{\mu}\right) e^{-\sqrt{\bar{s}^2-a^2}}}{\left(\frac{\bar{s}-a}{\sqrt{\bar{s}^2-a^2}} + \frac{1}{\mu}\right) e^{\sqrt{\bar{s}^2-a^2}} - \left(\frac{\bar{s}-a}{\sqrt{\bar{s}^2-a^2}} - \frac{1}{\mu}\right) e^{-\sqrt{\bar{s}^2-a^2}}} .$$

It can be seen that the expressions in the numerator and denominator of the expression given for Γ_V are entire functions of $\bar{S}$. The inverse Lapalace transform of the denominator gives the K-pulse $K(t)$ while the reflected pulse $\Gamma(t)$ is the inverse transform of the numerator, i.e.

$$e^{at}K(t) \overset{\mathcal{L}}{\leftrightarrow} K(\bar{s}) = e^a \left[\left(\frac{\bar{s}-a}{\sqrt{\bar{s}^2-a^2}} + \frac{1}{\mu}\right) e^{-(\bar{s}-\sqrt{\bar{s}^2-a^2})} - \left(\frac{\bar{s}-a}{\sqrt{\bar{s}^2-a^2}} - \frac{1}{\mu}\right) e^{-\sqrt{\bar{s}^2-a^2}} e^{-\bar{s}} \right] ,$$

and

$$e^{at}\Gamma(t) \overset{\mathcal{L}}{\leftrightarrow} R(\bar{s}) = e^a \left[\left(\frac{\bar{s}-a}{\sqrt{\bar{s}^2-a^2}} - \frac{1}{\mu}\right) e^{-(\bar{s}-\sqrt{\bar{s}^2-a^2})} - \left(\frac{\bar{s}-a}{\sqrt{\bar{s}^2-a^2}} + \frac{1}{\mu}\right) e^{-\bar{s}} e^{-\sqrt{\bar{s}^2-a^2}} \right] .$$

By using the tables for inverse transforms:

$$\frac{S}{\sqrt{s^2-a^2}} \; e^{-(S-\sqrt{s^2-a^2})} \leftrightarrow \delta(t) + \frac{a(t-1)}{\sqrt{t^2-2t}} \; I_1(a\sqrt{t^2-2t}) \; U(t) \; .$$

$$\frac{S}{\sqrt{s^2-a^2}} \; e^{-\sqrt{s^2-a^2}} \leftrightarrow \delta(t-1) + at \; \frac{I_1 \; (a\sqrt{t^2-1})}{\sqrt{t^2-1}} \; U(t-1) \; .$$

$$\frac{e^{-(S-\sqrt{s^2-a^2})}}{\sqrt{s^2-a^2}} \leftrightarrow I_0(a\sqrt{t^2-2t}) \; U(t) \; .$$

$$\frac{e^{-\sqrt{s^2-a^2})}}{\sqrt{s^2-a^2}} \leftrightarrow I_0(a\sqrt{t^2-1}) \; U(t-1) \; .$$

$$e^{-(S-\sqrt{s^2-a^2})} \leftrightarrow \delta(t) - \frac{aI_1(a\sqrt{t^2-2t})}{\sqrt{t^2-2t}} \; U(t) \; .$$

$$e^{-\sqrt{s^2-a^2})} \leftrightarrow \frac{aI_1(a\sqrt{t^2-1})}{\sqrt{t^2-1}} \; U(t-1) + \delta(t-1) \quad ,$$

we obtain

$$K(t) = e^{-K\left(\frac{1}{2\tau} - \frac{1}{2}\right)} \left[\left(1 + \frac{1}{\mu}\right)\right] \left\{ \delta(t) - \frac{\mu-1}{\mu+1} \; \delta(t-2\tau) \right.$$

$$+ \frac{K}{4\tau} \left(\frac{\mu}{1+\mu}\right)\left[\frac{\left(\frac{t}{\tau} -1\right) - \frac{1}{\mu}}{\sqrt{\left(\frac{t}{2\tau}\right) - \left(\frac{t}{2\tau}\right)^2}} \; J_1\left(K\sqrt{\left(\frac{t}{2\tau}\right) - \left(\frac{t}{2\tau}\right)^2}\right)\right.$$

$$\left. \left. - 2J_o\left(K\sqrt{\frac{t}{2\tau} - \left(\frac{t}{2\tau}\right)^2}\right) \right] \left[U(t) - U(t-2\tau)\right] \right\} \quad ,$$

and

$$R(t) = e^{-K\left(\frac{t}{2\tau} - \frac{1}{2}\right)} \left(1 - \frac{1}{\mu}\right) \left\{ \delta(t) - \frac{\mu+1}{\mu-1} \delta(t-2\tau) \right.$$

$$+ \frac{K}{4\tau} \left(\frac{-\mu}{1-\mu}\right) \left[\frac{\left(\frac{t}{\tau} - 1\right) + \frac{1}{\mu}}{\sqrt{\left(\frac{t}{2\tau}\right) - \left(\frac{t}{2\tau}\right)^2}} J_1\left(K\sqrt{\frac{t}{2\tau} - \left(\frac{t}{2\tau}\right)^2}\right)\right.$$

$$\left.\left. - 2J_0\left(K\sqrt{\left(\frac{t}{2\tau}\right) - \left(\frac{t}{2\tau}\right)^2}\right)\right] \left[U(t) - U(t-2\tau)\right] \right\}.$$

If we set $2\tau = 1$, the total duration of $R(t) = 1$ and $K(t) = 1$, then the normalized K-pulse is given by

$$K(t) = \delta(t) - e^{-K} \left(\frac{\mu-1}{\mu+1}\right) \delta(t-1)$$

$$+ \frac{K}{2} \left(\frac{\mu}{1+\mu}\right) e^{-kt} \left[\frac{(2t-1) - \frac{1}{\mu}}{\sqrt{t - t^2}} J_1\left(K\sqrt{t-t2}\right)\right.$$

$$\left. - 2J_0\left(K\sqrt{t-t^2}\right)\right] [U(t) - U(t-1)] ,$$

and the reflected pulse is

$$R(t) = \frac{\mu-1}{\mu+1} \delta(t) - e^{-K}\delta(t-1)$$

$$+ e^{-kt} \frac{K}{2} \left(\frac{\mu}{1+\mu}\right) \left[\frac{2t-1+\frac{1}{\mu}}{\sqrt{t-t^2}} J_1(K\sqrt{t-t^2}) \right.$$

$$\left. - 2J_0 (K\sqrt{t-t^2}) \right] [U(t) - U(t-1)] \, .$$

APPENDIX II

ELEMENT VALUES FOR DISTRIBUTED SHUNT CONDUCTANCE LINES

The relation between the R matrix and S matrix has been indicated in Sec. 2. The elements of the S matrix for the finite element model of the distributed line is given by

$$S_{11} = \frac{Y_c - Y_{in}}{Y_c + Y_{in}} .$$

$$S_{12} = S_{21} = 1+S_{11} ,$$

where Y_c is the characteristic admittance of the line. For a typical section of the line with a shunt conductance load G_n, $Y_{in} = Y_c + G_n$, thus

$$S_{11} = \frac{-G_n}{2Y_c + G_n} = \frac{-G_n/Y_c}{2 + G_n/Y_c} ,$$

and

$$S_{12} = \frac{2}{1 + \frac{G_n}{Y_c}} .$$

In the case of a line with distributed shunt conductance, we assume that the total shunt conductance G_{sh} equals to a specified fraction of the characteristic admittance, i.e., $G_{sh} = kYc$. Then, the element value G_n for a N section model is simply:

$$G_n = \frac{kYc}{N} \text{ for uniform lossy line,}$$

and

$$G_n = \frac{2kN}{N(N+1)} Yc \text{ for linearly tapered line.}$$

With the G_n given as above, it is straightforward to calculate the R-matrix elements described in Sec. 3.

APPENDIX III

ELEMENT VALUES FOR VARIABLE CHARACTERISTIC IMPEDANCE LINES

The approximate K-pulse and response waveforms for a line with continuously varying characteristic impedance are derived by using a model with N uniform line segment in cascade, the characteristic impedances of the discrete elements matching that of the line as a staircase function. For the special case arising when a non-uniform line is used to model transmission and reflection by a planar dielectric slab with a linearly varying dielectric constant ε_r, the dielectric constant ε_n for the n^{th} section of the N-section model is simply

$$\varepsilon_n = \frac{(\varepsilon_2 - \varepsilon_1)}{2N} (2n-1) + \varepsilon_1 \quad ,$$

where ε_1 is the dielectric constant corresponding to the entering line, and ε_2 is that corresponding to the exiting line. With ε_n given as above, the element for the R matrix can be readily determined as described in Sec. 4.

REFERENCES

(1) "Principles of Microwave Circuits", Chap. V, Sec. 5.18, Radiation Laboratory Series, 8, Montgomery, Dicke and Purcell, Ed., McGraw-Hill Book Company, 1948.

(2) "Acoustic, Electromagnetic and Elastic Wave Scattering - Focus on the T-matrix Approach", Varadan and Varadan, Ed., Pergamon Press, 1980.

(3) "Basic Theory of Waveguide Junctions and Introductory Microwave Network Analysis", Kerns and Beatty, Ed., Pergamon Press.

(4) "Fifth Annual Report 710816-12", December 1982, The Ohio State University ElectroScience Laboratory, Department of Electrical Engineering; prepared under Contract No. N00014-78-C-0049 for the Department of the Navy, Office of Naval Research, Arlington, Virginia 22217.

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